

미적분학 및 연습 I (MATH161) 제3차시험 (2011년 1학기)

학과:

학번:

이름:

1. (13 점)

(a) Find the area of the region that lies inside both curves

$$r = \sin 2\theta, \quad r = \sin \theta$$

(b) Find all points of intersection of those curves.

2. (12 점) If $\sum_{n=1}^{\infty} a_n$ converges, and if $1 > a_n > 0$ for all n , which of the following series are convergent? Explain.

$$\sum_{n=1}^{\infty} a_n^2, \quad \sum_{n=1}^{\infty} \frac{a_n}{1 - a_n}, \quad \sum_{n=1}^{\infty} \ln(1 - a_n)$$

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3. (13 점)

- (a) For which positive integers k is the following series convergent?

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$$

- (b) If k is a positive integer, find the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(n!)^k}{(kn)!} x^n$$

4. (12 점) Determine whether the following series converges or diverges

$$\sum_{k=1}^{\infty} \frac{\ln k}{k\sqrt{k}}$$

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5. (12 점) Test the following series for (a) absolute convergence and (b) conditional convergence

$$\frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} - \frac{1}{7} + \cdots + \frac{1}{3k+2} - \frac{1}{3k+3} - \frac{1}{3k+4} + \cdots$$

6. (13 점)

(a) Expand $f(x) = \ln \cos x$ in powers of x .

(b) Use Taylor polynomials to estimate \sqrt{e} to within 0.01.

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7. (13 점) Lines l_1 and l_2 in space are parallel to the vectors d_1 and d_2 , respectively. P is a point on l_1 and Q is a point on l_2 . If l_1 and l_2 are skew (i.e. nonintersecting and nonparallel) lines, express the distance between two lines using \overrightarrow{PQ} and $d_1 \times d_2$.

8. (12 점) Find the area of the surface generated by revolving one arch of the cycloid

$$x = a(t - \sin t), \quad y = a(1 - \cos t)$$

about the x -axis for $a > 0$.